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INTERIOR BALLISTICS

by

M. E. Serevryakov

State Printing House of the
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Moscow, 1949, 2nd Edition

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SECTION V - PHENOMENA ASSOCIATED WITH GAS DISCHARGE

GENERAL COMMENTS

The fundamental equations of internal ballistics permit the solution of the problem dealing with the motion of the projectile up to the instant at which the base of the shell traverses the muzzle face of the barrel.

However, the action of the gases continues or persists even after the projectile's departure from the bore. Both the projectile and the barrel experience for a certain period of time the so-called "after-effect" of the powder gases in the form of continued gas pressure exerted on both the projectile and the barrel. While it is possible to approximately determine the theoretical relation between the pressures acting on the base of the bore and the base of the projectile while the latter is travelling through the bore, this relationship will be entirely different after the projectile has left the bore. That portion of the gas which, upon leaving the bore, continues to exert a pressure on the projectile for a certain period of time, will be subjected to conditions differing sharply from the conditions to which the gases still remaining in the barrel are subjected.

The gases remaining in the barrel continue in their motion along the axis of the bore from the base of the barrel to its muzzle face, and upon leaving the bore commence to disperse somewhat from their basic direction of motion. The resulting reaction imparts additional acceleration to the recoiling barrel and the maximum speed of recoil obtains after the projectile's departure from the barrel. Almost the entire mass of gases generated by the charge participates in the reaction on the barrel. After the projectile's departure, only

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the central portion of the gases which had retained its axial motion along the bore continues to react on the base of the barrel; the greater portion of the gas either temporarily overtakes the projectile or is dissipated laterally in the form of a gas cloud. Inasmuch as the density of the gas discharge rapidly drops as the gases expand, the gases lose speed very rapidly and fall behind the projectile. Nevertheless, the projectile's speed will continue to increase somewhat even while the gases are lagging behind it. During this period the fuze mechanisms are triggered and go into action. A study of the projectile's motion after its departure from the bore constitutes one of the problems of internal ballistics.

The period of gas after-action, or the third period, which is a direct continuation of the shot phenomenon accompanied by gas discharge from the gun barrel, constitutes a problem involving the derivation of special fundamental relations for its solution. This, in turn, requires a knowledge of the fundamental laws of gas discharge.

General relations of gas dynamics are also required for the solution of special problems arising from the complex structure of artillery weapons and with the appearance of new systems, in which the gases are discharged from ports of various types.

Such systems may include:

1) Automatic weapons in which the gases are discharged from the bore before the projectile's departure, or guns with muzzle attachments.

2) Weapons with separate powder chambers with gas discharge through a single or multiple nozzles (weapons operating on the principle of gas dynamics or hydrodynamics).

3) Recoilless guns, in which the gases are discharged through an opening in the breechblock.

4) Muzzle brakes, in which the gases are discharged through passageways laterally, and by exerting pressure on the walls of these passageways brake the recoil and reduce the recoiling speed.

5) Mine throwers, in which a portion of the gases overtakes the mine while it is moving through the bore; special mine throwers with a remotely controlled valve, in which a portion of the gases is discharged through the valve and does not participate in the action exerted on the mine; furthermore, mine throwers are provided with means for discharging the gas from the inner chamber, containing the tail cartridge and the main charge, into a chamber containing additional charge.

6) Special manometric bombs with nozzles used for studying the powder burning phenomenon by means of gas discharge through a nozzle.

7) Rocket chambers.

In all the systems mentioned above, the gas is discharged through ports of varying shapes and sizes under high pressure. In order to establish the proper relations, ~~taking into account all of these phenomena and their peculiarities~~, use must be made of the general formulas relating to gas discharge. Therefore, all the derived relations constitute first approximations only and require further refinement on the basis of test data obtained during firing of weapons.

CHAPTER 1 - GENERAL INFORMATION ON GAS DYNAMICS

1. RATE OF GAS DISCHARGE

Using the designations:

U, V, W - gas velocities projected on the coordinate axes;

X, Y, Z - projections of external volume forces on the same axes;

ρ' - density of a unit mass of gas;

p - pressure,

then the fundamental Euler's equation of hydrodynamics with respect to the x-axis will read as follows:

$$\frac{dU}{dt} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} = X - \frac{1}{\rho'}\frac{\partial p}{\partial x} \qquad (77)$$

and will be analogous in character with respect to the other axes.

We shall consider the gas discharge as a single-dimensional motion in the direction of the x-axis, due to pressure difference, in the absence of external forces (X = 0):

$$\frac{dU}{dt} = \frac{\partial U}{dt} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho'}\frac{\partial p}{\partial x}. \qquad (78)$$

The values of p and U for a stabilized motion do not depend on t and are functions of x only. In this case $\frac{\partial U}{\partial t} = 0$:

$$U\frac{dU}{dx} = -\frac{1}{\rho'}\frac{dp}{dx}$$

or

$$-\frac{dp}{\rho'} = UdU = d\,\frac{U^2}{2}.$$

Replacing the mass density ρ' by gravimetric density ρ and bearing in mind that $\rho = 1/w$, where w is the specific volume of gas, we get:

$$-w\,dp = d\,\frac{U^2}{2g}. \tag{79}$$

If we designate the pressure, specific volume and velocity in the vessel from which the gases are discharged by p_1, w_1, U_1, respectively, then, upon integrating expression (79), we will get

$$-\int_{p_1}^{p} w\,dp = \int_{p}^{p_1} w\,dp = \frac{U^2 - U_1^2}{2g}. \tag{80}$$

In order to integrate the left side of equation (80), the dependence of w on p for the process taking place in the gas must be known. We shall consider the polytropic process, of which the adiabatic process constitutes a particular case:

$$pw^k = p_1 w_1^k = \text{const},$$

whence

$$w = \frac{1}{\rho} = w_1 \frac{p_1^{1/k}}{p^{1/k}}.$$

Substituting this expression in equation (80) and integrating, we get:

$$w_1 p_1^{1/k} \int_{p_1}^{p} \frac{dp}{p^{1/k}} = \frac{U^2 - U_1^2}{2g};$$

$$w_1 p_1^{1/k} \frac{k}{k-1}\left(p_1^{k-1/k} - p^{k-1/k}\right) = \frac{k}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right] =$$

$$= \frac{U^2 - U_1^2}{2g},$$

whence

$$U = \sqrt{U_1^2 + \frac{2gk}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right]}. \tag{81}$$

This is Saint-Venant's formula.

If we assume that $U_1 = 0$ when the discharge is from a very large vessel, we shall get an expression for the velocity of the gas discharged into space with a pressure p from the vessel under pressure p_1.

$$U = \sqrt{\frac{2gk}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right]}. \tag{82}$$

The maximum velocity will obtain when the discharge is into vacuum, when $p = 0$; we will have:

$$U_{max} = \sqrt{\frac{2gk}{k-1} p_1 w_1}.$$

however, we know from physics that $\sqrt{gkp_1 w_1} = \sqrt{gkRT_1} = C_1$ is the speed of sound in gas, corresponding to the given condition p_1 and w_1 or T_1 of the gases present in the vessel from which the gases

are discharged. Hence,

$$U_{max} = \sqrt{\frac{2}{k-1}}\, C_1.$$

GRAPHIC NOT REPRODUCIBLE

Fig. 112 - Dependence of Velocity of Gas Discharge on p/p_1.

Substituting this expression in equation (82), we get:

$$U = U_{max}\sqrt{1 - \left(\frac{p}{p_1}\right)^{k-1/k}} = C_1 \sqrt{\frac{2}{k-1}\left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right]}.$$

The dependence of the velocity of discharge on pressure p or the ratio p/p_1 is depicted in fig. 112. When $p = 0$, $U = U_{max}$. As the back pressure p increases, U decreases and has a point of inflexion ($U_{cr.}$, $x_{cr.}$); and when $p/p_1 = 1$ it becomes zero, i.e., the discharge ceases.

We shall discuss the values $x_{cr.}$ and $U_{cr.}$ later in the text.

2. THE GRAVIMETRIC CONSUMPTION OF GASES G_{sec} PER SECOND

If the stream of gases discharged under a high pressure has a velocity U, density $\rho = 1/w$ and a cross-sectional area s, the gas consumption per second will be:

$$G_{sec} = sU\rho = s\rho\sqrt{\frac{2gk}{k-1}p_1 w_1\left[1-\left(\frac{p}{p_1}\right)^{k-1/k}\right]}. \quad (83)$$

Since for a polytropic process

$$\rho = \frac{1}{w} = \frac{1}{w_1}\left(\frac{p}{p_1}\right)^{1/k},$$

then, substituting this expression in the formula for determining the expenditure per second, we get:

$$G_{sec} = s\frac{1}{w_1}\left(\frac{p}{p_1}\right)^{1/k}\sqrt{\frac{2gk}{k-1}p_1 w_1\left[1-\left(\frac{p}{p_1}\right)^{k-1/k}\right]} =$$

$$= s\sqrt{\frac{2gk}{k-1}\frac{p_1}{w_1}}\sqrt{\left(\frac{p}{p_1}\right)^{2/k} - \left(\frac{p}{p_1}\right)^{k+1/k}} \quad (84)$$

(Tseiner's formula).

Designating the p/p_1 ratio by x and the constant $\sqrt{\frac{2gk}{k-1}\frac{p_1}{w_1}}$ by a_1, we find:

$$G_{sec} = a_1 s\sqrt{x^{2/k} - x^{k+1/k}} = a_1 s f(x).$$

When the motion is steady, the consumption per second is constant; therefore,

$$sf(x) = s\sqrt{x^{2/k} - x^{k+1/k}} = \frac{G_{sec}}{a_1} = \text{const}$$

and

$$s = \frac{const}{f(x)},$$

i.e., the cross section of the stream varies inversely with the change $f(x)$ depending on p/p_1.

Investigations show that $f(x)$ has a maximum when the value $\frac{p_{cr.}}{p_1} = x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1}$; therefore, the cross-sectional area s will be minimum at this value. The pressure ratio $p_{cr.}/p_1$ at which the cross section of the stream is minimum and the flow through a unit cross-sectional area is maximum is called the critical pressure ratio, and the cross section is called the critical cross section.

The value $x_{cr.}$ depends on the polytropic index, though to a small degree only. The following table gives the values of $x_{cr.}$ with relation to k (Table 27).

Table 27

k	1.41	1.30	1.25	1.20	1.10
$x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1}$	0.527	0.546	0.555	0.565	0.585

Substituting the value $x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1}$ in formula (82) for the discharge velocity, we get the following expression for the "critical" gas velocity:

$$U_{cr.} = \sqrt{\frac{2gk}{k+1}p_1 w_1} = \sqrt{\frac{2}{k+1}}c_1.$$

This value approaches the speed of sound in a gas located in a vessel, from which the discharge takes place, and whose equation of

state is determined by the values p_1 and w_1.

Since from the adiabatic equation $p_1 w_1 = p_{cr.} w_{cr.} \frac{k+1}{2}$, the expression for critical velocity will take on the form:

$$U_{cr.} = \sqrt{gkp_{cr.} w_{cr.}} = c_{cr.},$$

i.e., the critical velocity at the minimal cross section at the point of critical pressure equals the velocity of sound, corresponding to the state of gas at this critical pressure. This velocity is shown in fig. 112 in the form of segment $U_{cr.}$ at $x_{cr.}$.

Having determined the critical pressure and velocity of the gases, we shall now find the consumption through the smallest cross section (which we shall designate by s_m). To do this, we substitute in the right side of formula (84) the value

$$\frac{p_{cr.}}{p_1} = x_{cr.} = \left(\frac{2}{k+1}\right)^{k/k-1};$$

$$G_{sec} = s_m (x_{cr.})^{1/k} \sqrt{\frac{2gk}{k-1} \frac{p_1}{w_1} \left[1 - x_{cr.}^{k-1/k}\right]} =$$

$$= s_m \left(\frac{2}{k+1}\right)^{1/k-1} \sqrt{\frac{2gk}{k+1} \frac{p_1}{w_1}} = K_0 s_m \sqrt{\frac{p_1}{w_1}}.$$

Here the coefficient $K_0 = \sqrt{\frac{2gk}{k+1}} \left(\frac{2}{k+1}\right)^{1/k-1}$ is a constant which, depending on the exponent k, varies within small limits in accordance with Table 28 ($g = 98.1$ dm/sec^2).

Table 28

k	1.25	1.20	1.15	1.10
$\sqrt{\frac{2gk}{k+1}\left(\frac{2}{k+1}\right)^{1/k-1}} = K_0$	6.518	6.424	6.325	6.224

For the vessel in which the powder is burned, the expression for G_{sec} can be presented differently, by multiplying and dividing the expression under the root by p_1. Replacing, approximately, $p_1 w_1 = RT_1$ by f, we get

$$G_{sec} = \frac{K_0}{\sqrt{f}} s_m p_1 = As_m p_1, \qquad (85)$$

where $A = \frac{K_0}{\sqrt{f}}$ is a constant depending on the nature of the gases and their temperature, inasmuch as k is a function of the temperature;

s_m is the minimal cross section of the gas stream, which may be assumed to be an orifice with rounded edges or the minimum cross section in the Laval nozzle;

p_1 is the pressure at which the gases are discharged from the vessel.

Coefficient A, characterizing the consumption of gas at $s = 1$ and $p_1 = 1$ is measured in (sec^{-1}) and varies with f and k.

The coefficient A was first introduced by V.M. Trofimov who assumed $A = 0.007$ for pyroxylin powders and $A = 0.006$ for nitroglycerine powder.

Actually, when gas is discharged from a vessel, even in the case where powder is burned in the vessel, the temperature T inside the

vessel will be lower than T_1, and the value $p_1 w_1 = RT$, where $T < T_1$. Hence, it would be more correct to state:

$$G_{sec} = \frac{K_0}{\sqrt{p_1 w_1}} s_m p_1 = \frac{K_0}{\sqrt{RT}} s_m p_1 = \frac{K_0}{\sqrt{f}\sqrt{\tau}} s_m p_1 = \frac{A}{\sqrt{\tau}} s_m p_1,$$

where $\tau = T/T_1$ (see below).

Table 29

f \ k	1.1	1.2	1.3
1,000,000	0.00622	0.00642	0.00661
900,000	0.00656	0.00677	0.00697
850,000	0.00675	0.00695	0.00717
800,000	0.00695	0.00718	0.00739

3. FULL GAS CONSUMPTION

The full gas consumption Y over a period t can be obtained from the expression

$$Y = \int_0^t G_{sec}\, dt.$$

We proved earlier that

$$G_{sec} = A s_m p_1,$$

where p_1 - pressure in the vessel from which gas is discharged;

s_m - cross-sectional area of opening or orifice through which the gas flows.

If we apply this formula to a bomb with a nozzle, in which

pressure p is developed when the powder is burned, then

$$G_{sec} = As_{m}p,$$

where $p = f(t)$,

$$Y = As_{m}\int_{0}^{t} pdt = As_{m}I,$$

but $I = \frac{e}{u_1}$.

During the entire period that the powder is burned $\int_{0}^{1} pdt = I_K$, and hence the full consumption during the entire period of powder burning will be

$$Y_K = As_{m}I_K = As_{m}\frac{e_1}{u_1}.$$

When the cross section s_m of the nozzle, the nature of the powder gases and their temperature [$A = f(k, f)$], the thickness of the powder and its rate of burning are known, this formula enables us to compute in advance the consumption of powder by weight during the period the powder is burned in the chamber or in a bomb with a nozzle. This formula has been satisfactorily confirmed in bomb tests, in which the start of burning of cylindrical grains with very narrow perforations - 1 to 3 mm in diameter, at pressures p_m = 2000-2500 kg/cm^2 has been investigated.

4. THE DEPENDENCE OF GAS PRESSURE ON THE CROSS SECTION OF THE STREAM(*)

If the gases are discharged through a tapered diverging nozzle, the pressure in the direction of flow will decrease, whereas the velocity of discharge will increase. The pressure magnitudes at various sections can be found from the equation of continuity, because $G_{cr.} = G_x$, where G_x is the flow through section s_x. The equation of continuity will be written in the form:

$$\frac{s_m U_{cr.}}{w_{cr.}} = \frac{s_x U_x}{w_x},$$

but

$$\frac{s_m U_{cr.}}{w_{cr.}} = s_m K_0 \sqrt{\frac{p_1}{w_1}} = s_m \left(\frac{2}{k+1}\right)^{1/k-1} \sqrt{\frac{2gk}{k+1} \frac{p_1}{w_1}},$$

and according to formula (84)

$$\frac{s_x U_x}{w_x} = s_x \sqrt{\frac{2gk}{k-1} \frac{p_1}{w_1}} \sqrt{x^{2/k}\left(1 - x^{k-1/k}\right)}.$$

Equating the right sides of these equations and assuming that $k = \text{const}$ from one section to another, and reducing by $\sqrt{2gk \frac{p_1}{w_1}}$, we get:

(*) The derivation and numerical data are taken from the book by Prof. I.P. Grave, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics). Pyrodynamics, 3rd Edition, p. 217.

$$s_x x^{1/k}\sqrt{\frac{1-x^{k-1/k}}{k-1}} = s_m\left(\frac{2}{k+1}\right)^{1/k-1}\sqrt{\frac{1}{k+1}},$$

whence

$$\frac{s_x}{s_m} = \left(\frac{2}{k+1}\right)^{1/k-1}\frac{\sqrt{\frac{k-1}{k+1}}}{x^{1/k}\sqrt{1-x^{k-1/k}}}. \tag{86}$$

This equality gives the dependence of the relative pressure $\bar{x}$ in the region back of the minimum cross section on the relative cross section of the nozzle s_x/s_m. Upon assigning values of x for various values of k, a table can be compiled for the values of s_x/s_m, according to which an inverse problem can be solved: i.e., the value of the ratio $x = p/p_1$ (Table 30) can be found from the ratio of the cross section of the flow at a given point to its minimal cross section.

Table 30 - Values of s_x/s_m for Various x and k

k \ x		1/2	1/3	1/4	1/5	1/6	1/10	1/15	1/20
1.1	1	1.018	1.180	1.373	1.569	1.762	2.500	3.364	4.180
1.2	1	1.010	1.143	1.309	1.477	2.640	2.260	2.967	3.625
1.25	1	1.007	1.128	1.282	1.438	1.590	2.162	2.802	3.405
1.3	1	1.005	1.115	1.258	1.404	1.545	2.075	2.670	3.214
1.4	1	1.002	1.093	1.218	1.346	1.470	1.931	2.440	2.900

Example. Determine how much the cross section of the stream must be increased in order to obtain a 10-fold pressure decrease (at k = 1.25). At k = 1.25 and x = 1/10, we get $s_x/s_m = 2.162$.

5. EXPRESSION FOR DETERMINING THE REACTION PRESSURE DEVELOPED DURING GAS DISCHARGE THROUGH AN OPENING IN THE WALL OF THE VESSEL (TRACTIVE FORCE)

Let us assume (fig. 113) that a gas in a closed vessel is under pressure p. To each element of the surface s there is applied a force sp or $s(p - p_a)$, where p_a is atmospheric pressure. The velocity of the gas inside the vessel is $U_1 = 0$. When the opening of area s is opened, the gases will be discharged through it, and the vessel will be subjected to a reacting force composed of the following components:

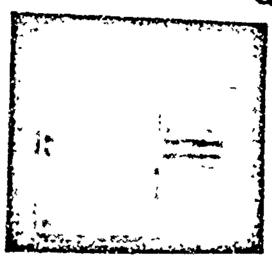

Fig. 113 - Diagram of Gas Discharge and the Reaction Pressure

1) The force $R' = s(p - p_a) \approx sp$, acting in all directions before the orifice is opened, and reacting in a direction opposite to the flow of the gases when a portion of the wall of area s disappears.

2) The force R", originating in consequence of the gas discharge through orifice s under the action of internal forces and determined on the basis of the mechanics theory concerned with momentum and force impulse.

The elementary mass dm discharged through area s during time dt acquires a velocity U and an increment of momentum dmU; it creates a force impulse R"dt in the reverse direction:

$$R''dt = dmU = \frac{\rho}{g} sUdtU = \frac{G_{sec}}{g} dtU,$$

whence

$$R'' = \frac{G_{sec}}{g} U.$$

The full reaction pressure or the tractive force originating upon the discharge of gas through opening s will be expressed by the formula:

$$R = R' + R'' = \frac{G_{sec}}{g} U + sp.$$

It is assumed thereby that the gas velocity inside the vessel is $U_1 = 0$ and that hence no change in gas momentum takes place in a vessel of a sufficiently large capacity.

If we apply this formula to the discharge opening of a diverging tapered nozzle of cross section s_a, whereby $p = p_a$ and $U = U'_a$, then

$$R = \frac{G_{sec}}{g} U_a + s_a p_a.$$

Replacing G_{sec}, U_a, s_a and p_a by values relating to the minimum cross section s_m and internal pressure p_1, we get, on the basis of formulas (82) and (86)(*):

(*) I.P. Grave, "PIRODINAMIKA" (Pyrodynamics), Part III.

$$R = \frac{G_{sec} \cdot U_a}{g} + p_a s_a =$$

$$= \frac{s_m}{g} \sqrt{gk} \left(\frac{2}{k+1}\right)^{k+1/k-1} \sqrt{\frac{p_1}{w_1}} \sqrt{\frac{2gk}{k-1}} \sqrt{p_1 w_1} \sqrt{1 - \left(\frac{p_a}{p_1}\right)^{k-1/k}} +$$

$$+ \frac{p_a}{p_1} p_1 \frac{s_a}{s_m} s_m = s_m \left[k \left(\frac{2}{k+1}\right)^{k/k-1} \sqrt{\frac{k+1}{k-1}}\, p_1 \sqrt{1 - x_a^{k-1/k}} + \right.$$

$$\left. + \frac{x_a p_1 \left(\frac{2}{k+1}\right)^{1/k-1} \sqrt{\frac{k-1}{k+1}}}{x_a^{1/k} \quad 1 - x_a^{k-1/k}} \right] =$$

$$= k x_m \sqrt{\frac{k+1}{k-1}} \sqrt{1 - x_a^{k-1/k}} \left[1 + \frac{k-1}{2k} \frac{x_a^{k-1/k}}{1 - x_a^{k-1/k}} \right] p_1 s_m. \qquad (87)$$

Only the k, x_a, p_1 and s_m values enter this expression. It may be thus concluded that the value of R is proportional to pressure p_1 inside the vessel and the area of smallest cross section s_m; it depends on the exponent k and is determined by the degree of divergence of the nozzle which, in turn, depends on the ratio s_a/s_m. Formula (87) can be presented in an abbreviated form:

$$R = \zeta s_m p_1,$$

assuming that

$$\zeta = k \left(\frac{2}{k+1}\right)^{k/k-1} \sqrt{\frac{k+1}{k-1}} \sqrt{1 - x_a^{k-1/k}} \left[1 + \frac{k-1}{2k} \frac{x_a^{k-1/k}}{1 - x_a^{k-1/k}}\right]. \tag{88}$$

The coefficient ζ for a given nozzle depends on k only; it depends on the nature of the powder only to the extent that it determines the value of k; it does not depend on the charging density, nor on the value of p.

Langevin calls this coefficient the propulsive action coefficient. In the absence of a nozzle, and if only an opening were present in the wall, then, at

$$x_a = x_m = \left(\frac{2}{k+1}\right)^{k/k-1}$$

we would obtain

$$\zeta_0 = k \left(\frac{2}{k+1}\right)^{k/k-1} \left(1 + \frac{k-1}{2k} \frac{\frac{2}{k+1}}{\frac{k-1}{k+1}}\right) =$$

$$= (k+1) \left(\frac{2}{k+1}\right)^{k/k-1} = (k+1)x_m. \tag{89}$$

If the nozzle were infinitely large and permitted infinite divergence, and if the outside pressure were disregarded (in other words, if the discharge were into vacuum), then:

$$\zeta_{max} = k \sqrt{\frac{k+1}{k-1}} \left(\frac{2}{k+1}\right)^{k/k-1} = k \sqrt{\frac{k+1}{k-1}} x_m .$$

The following table gives the dependence of coefficient ζ on the ratio between the discharge opening diameter d_a and the diameter of minimum cross section (Table 31).

Table 31

$\frac{d_a}{d_m}$	1	2	3	4	5	6
ζ	1.24	1.62	1.72	1.80	1.86	1.89
$\frac{s_a}{s_m}$	1	4	9	16	25	36

It can be seen from the table that as the outfit diameter of the nozzle increases, the reaction pressure increases rapidly at first and then slower and slower, approaching asymptotically the value ζ_{max}. In rocket shells it is customary to take $\frac{d_a}{d_m} \ngtr 3$ in order not be make the shell unnecessarily heavy. The coefficient ζ changes very little with the change of k.

6. FUNDAMENTAL FORMULAS

Thus, as a result of applying the laws of gas dynamics, the following relations have been established.

Gas discharge velocity:

$$U = \sqrt{\frac{2gk}{k-1} p_1 w_1 \left[1 - \left(\frac{p}{p_1}\right)^{k-1/k}\right]} ,$$

where p_1 and w_1 are the pressure and unit volume of gases in the vessel from which the discharge takes place.

Gas consumption through cross section s_m per second:

$$G_{sec} = As_m p_1,$$

where A - a coefficient depending on the nature of the powder (f and $k = 1 + \Theta$);

$A \approx 0.007$ for pyroxylin powders;

$A \approx 0.0065$ for nitroglycerine powders.

The gas consumption in time t is

$$Y = \int_0^t G_{sec} dt = As_m \int_0^t p_1 dt.$$

If the gases are formed in the vessel in consequence of powder burning, then

$$\int_0^t p dt = \frac{e}{u_1}; \quad Y = As_m \frac{e}{u_1};$$

the full gas consumption during the powder burning period is

$$Y_K = As_m \frac{e_1}{u_1}.$$

The reaction force of the discharged gases is

$$R = \frac{G_{sec}}{g} U + sp_1 = \zeta s_m p_1,$$

where ζ is given in the above table and is practically independent of the coefficient $k = 1 + \Theta$. Thus the basic values G_{sec}, Y and R are very simple functions of ballistic elements and of the gas pressure

Gas consumption through cross section s_m per second:

$$G_{sec} = As_m p_1,$$

where A - a coefficient depending on the nature of the powder (f and $k = 1 + \Theta$);

$A \approx 0.007$ for pyroxylin powders;

$A \approx 0.0065$ for nitroglycerine powders.

The gas consumption in time t is

$$Y = \int_0^t G_{sec}dt = As_m \int_0^t p_1 dt.$$

If the gases are formed in the vessel in consequence of powder burning, then

$$\int_0^t pdt = \frac{e}{u_1}; \quad Y = As_m \frac{e}{u_1};$$

the full gas consumption during the powder burning period is

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The reaction force of the discharged gases is

$$R = \frac{G_{sec}}{g} U + sp_1 = \zeta s_m p_1,$$

where ζ is given in the above table and is practically independent of the coefficient $k = 1 + \Theta$. Thus the basic values G_{sec}, Y and R are very simple functions of ballistic elements and of the gas pressure

characteristics p in the vessel and of the pressure impulse at a given instant:

$$\int_0^t pdt = I = \frac{e}{u_1}$$

and at the end of burning

$$I_K = \int_0^{t_K} pdt = \frac{e_1}{u_1}.$$

In some cases the movement of gas inside the vessel from which it is discharged cannot be disregarded, as, for example, in the case of gases discharged from the bore of a gun, wherein the gas velocity varies linearly from zero at the base of the chamber to U_A at the face of the muzzle. In such a case an additional term will be added to the two components of the reaction pressure (force) depicting the change of gas momentum in the bore of the barrel:

$$R = \frac{G_{sec}}{g} U + sp_1 + \frac{dI}{dt}.$$

When the gas velocity changes linearly:

$$\frac{dI}{dt} = \mu \frac{U}{2} = \frac{\omega}{g} \frac{v_{Aa}}{2}.$$

CHAPTER 2 - THE APPLICATION OF BASIC FORMULAS OF GAS DISCHARGE

1. GAS DISCHARGE FROM A VESSEL OF SPECIFIC VOLUME

Say, a volume W_0 contains ω kg of gas. The initial state of the gas is characterized by the values p_1, T_1, $w_1 = \frac{W_0}{\omega}$. It is necessary to determine the law governing the pressure and temperature drop as a function of time. As in the case of the general theory of discharge, we shall consider the process an adiabatic one. Then

$$\left(\frac{p}{p_1}\right)^{1/k} = \frac{w_1}{w},$$

but

$$w_1 = \frac{W_0}{\omega}, \quad w = \frac{W_0}{\omega - \int_0^t G_{sec} dt}.$$

Therefore,

$$\left(\frac{p}{p_1}\right)^{1/k} = \frac{\omega - \int_0^t G_{sec} dt}{\omega} = 1 - \frac{\int_0^t G_{sec} dt}{\omega}, \tag{90}$$

whereby

$$G_{sec} = sK_0 \sqrt{\frac{p}{w}}.$$

Inasmuch as

$$\frac{p}{w} = p_1 \frac{p}{p_1} \frac{1}{w_1} \left(\frac{p}{p_1}\right)^{1/k} = \frac{p_1}{w_1} \left(\frac{p}{p_1}\right)^{k+1/k},$$

$$G_{sec} = sK_0 \sqrt{\frac{p_1}{w_1}} \left(\frac{p}{p_1}\right)^{k+1/2k}.$$

Differentiating expression (90), we get:

$$\frac{1}{k} \frac{p^{1-k/k}}{p_1^{1/k}} dp = - \frac{G_{sec}}{\omega} dt = - \frac{sK_0}{\omega} \sqrt{\frac{p_1}{w_1}} \left(\frac{p}{p_1}\right)^{k+1/2k} dt.$$

Separating the variables and integrating:

$$\left(\frac{p}{p_1}\right)^{1-k/k} \left(\frac{p_1}{p}\right)^{k+1/2k} \frac{dp}{p_1} = - \frac{ksK_0}{\omega} \sqrt{\frac{p_1}{w_1}} dt = - bdt,$$

where

$$b = \frac{ksK_0}{\omega} \sqrt{\frac{p_1}{w_1}};\quad x^{1-3k/2k} dx = - bdt;\quad \int_1^x x^{1-3k/2k} dx = -bt$$

or

$$\frac{2k}{k-1} \left[1 - \frac{1}{x^{k-1/2k}}\right] = - \frac{ksK_0}{\omega} \sqrt{\frac{p_1}{w_1}} t = -bt.$$

This enables us to find the duration of discharge when the pressure drops from the initial value p_1 to the given value p.

$$t = \frac{1}{B'}\left[\frac{1}{x^{k-1/2k}} - 1\right],$$

where

$$B' = \frac{k-1}{2} s_m \frac{K_0}{\omega}\sqrt{\frac{p_1}{\omega_1}} \text{ and } K_0 = \sqrt{\frac{2gk}{k+1}}\left(\frac{2}{k+1}\right)^{1/k-1}.$$

This relationship is valid until the ratio between the outside and inside pressures becomes equal to the critical value. When the discharge is into the atmosphere

$$x_{cr.} = \frac{p_a}{p_{cr.}},\ p_{cr.} = \frac{p_a}{x_{cr.}} \approx 1.8\ \text{kg/cm}^2.$$

The full time of discharge is

$$t_{fl} = \frac{1}{B'}\left[\frac{1}{x_{cr.}^{k-1/2k}} - 1\right]. \tag{91}$$

These formulas show that the length of discharge up to a given pressure is inversely proportional to the cross section of the nozzle s_m. Solving the formula with respect to $p = p_1 x$, we get the relative pressure change as a function of time:

$$p = \frac{p_1}{(1 + B't)^{2k/k-1}}. \tag{92}$$

If the values of t are given, p can be calculated and a p, t curve can be constructed. The larger the cross section of the opening and the greater the value of p_1, the more rapid will be the relative pressure drop within the vessel. Inasmuch as

$$w = w_1 \left(\frac{p_1}{p}\right)^{1/k} \quad \text{and} \quad \frac{T}{T_1} = \left(\frac{w_1}{w}\right)^{k-1},$$

$$w = w_1 (1 + B't)^{2/k-1} \tag{93}$$

and

$$T = \frac{T_1}{(1 + B'T)^2}. \tag{94}$$

A comparison of formulas (94) and (92) shows that the gas temperature drop inside the vessel occurs much slower than the pressure drop.

2. GAS DISCHARGE FROM THE BORE OF A GUN AFTER THE PROJECTILE LEAVES THE GUN

Applying the relations obtained above to the gases discharged from the barrel bore after the projectile leaves the latter, we will have the following: prior to the start of discharge the barrel will contain ω kg of gas, so that the specific gas volume within the entire volume of the bore $W_0 + sl_A$ will be

$$w_A = \frac{W_0 + sl_A}{\omega} = \frac{W_{KH}}{\omega} = \frac{\Lambda_A + 1}{\Delta}.(*)$$

(*) Subscripts A and KH stand for "muzzle" and "bore," respectively.

The pressure at the atart of discharge equals the muzzle pressure $p_Д$, the gas temperature is $T_Д$, the cross-sectional area of the flow equals the cross-sectional area of the bore s.

Designating by B' the constant parameter

$$B' = \frac{k-1}{2}K_0 \sqrt{\frac{p_Д}{w_Д}} \frac{s}{\omega} = \frac{k-1}{2}K_0 \frac{s}{\omega} \sqrt{\frac{p_Д \Delta}{\Delta_Д + 1}}$$

for pressure, gas temperature and the time of gas discharge, we will obtain the following expressions:

$$p = \frac{p_Д}{(1 + B't)^{2k/k-1}};$$

$$T = \frac{T_Д}{(1 + B't)^2};$$

$$t = \frac{1}{B'}\left[\left(\frac{p_Д}{p}\right)^{k-1/2k} - 1\right].$$

All of these relations are valid while $x > x_{cr.} = 0.565\text{-}0.545$, i.e., up to a pressure of $p_{cr.} = \frac{p_a}{x_{cr.}} \approx 1.8$ kg/cm^2. The total duration of the after-action (or after-effect) of gases on the gun mount is determined by the following formula:

$$t_П = \frac{1}{B'}\left[\left(\frac{p_Д}{1.8}\right)^{k-1/2k} - 1\right].$$

Example. Given a 76-mm gun, $s = 0.4693$ dm^2, $\Lambda_Д = 9.0$, $\omega = 1.080$ kg,

$\Delta = 0.70,\ p_{д} = 600\ \text{kg/cm}^2,\ k = 1.2,\ K_0 = 6.424.$

$$B' = \frac{0.2}{2}\, 6.424\, \frac{0.4693}{1.080} \sqrt{\frac{60000 \cdot 0.70}{10}} = 0.2793 \cdot 64.8 = 18.10;$$

$$\frac{p_{д}}{1.8} = \frac{600}{1.8} = 333.3; \quad \log\left(\frac{p_{д}}{1.8}\right) = 2.523; \quad \frac{k-1}{2k} = \frac{1}{12};$$

$$\frac{1}{12} \log\left(\frac{p_{д}}{1.8}\right) = 0.2103; \quad \left(\frac{p_{д}}{1.8}\right)^{1/12} = 1.623; \quad t_{п} = \frac{0.623}{18.10} =$$

$$= 0.03443 \text{ sec.}$$

The discharge time until the pressure is 20 atm

$$t_{20} = \frac{1}{18.10}\left[\left(\frac{600}{20}\right)^{1/12} - 1\right] = \frac{0.327}{18.10} = 0.01807 \text{ sec.}$$

i.e., the time is almost one-half the full period of discharge down to $p_1 = 1.8$.

If $T_{д} = 0.70 \cdot T_1 = 0.70 \cdot 2800 = 1960^\circ K$, then

$$T_{п} = \frac{1960}{(1 + 18.1 t_{п})^2} = \frac{1960}{(1 + 0.623)^2} = \frac{1960}{2.635} = 744^\circ K = 471^\circ C.$$

3. THE AFTER-ACTION OF GASES ON THE GUN MOUNT

The relations introduced in Section 2 enable us to investigate the after-action of gases on the recoiling parts after the projectile leaves the gun, and, in particular, to determine the highest velocity

of recoil, necessary for the design of the gun mount.

GRAPHIC NOT REPRODUCIBLE

Fig. 114 - Velocity of Recoil During the Period of After-Action.

The reaction force R, arising as a result of gas discharging from the bore of the gun, imparts an added impulse to the recoiling parts and increases the velocity of recoil.

The action of the gases ceases at the end of their discharge, at which time the recoiling parts attain their maximum velocity V_{max}.

The curves in fig. 114 depict the gas pressure $p_{дн}$ on the base of the bore and the velocity of the recoiling parts V. $V_{д}$ corresponds to the instant the projectile leaves the bore, V_{max} corresponds to the period of after-action, $t_{д}$ is the time of recoil prior to the projectile's departure from the bore, $t_{п}$ is the period of gas after-action.

If the recoil is free, the relation between the velocity of recoil V and the velocity of the projectile (absolute) v_a is expressed by the formula:

$$V = \frac{q + \frac{1}{2}\omega}{Q_0 + \frac{1}{2}\omega} v_a \approx \frac{q + \frac{1}{2}\omega}{Q_0} v_a,$$

because $\frac{1}{2}\omega$ is small compared with Q_0.

Prior to the instant the projectile leaves the bore

$$V_A = \frac{q + \frac{1}{2}\omega}{Q_0} v_{A.a},$$

where $v_{A.a}$ is the absolute velocity of the projectile at the instant it leaves the gun. The recoil velocity increment $\Delta V = V_{max} - V_A$ is obtained as a result of the action produced by the reaction force impulse developed during gas discharge:

$$\frac{Q_0}{g} V_{max} - \frac{Q_0}{g} V_A = \int_0^{t_n} R dt. \qquad (95)$$

When the recoil is subjected to a braking effect

$$\frac{Q_0}{g} V_{max} - \frac{Q_0}{g} V_A = \int_0^{t_n} R dt - \int_0^{t_n} F dt,$$

where F is the resultant of the forces braking the recoil.

The problem dealing with the force R under conditions of powder gas discharge from the barrel bore has been considered in considerable detail in a series of special texts.

We shall assume some of the simplest allowances, to wit: 1) the cross section s of the barrel bore is the critical one; 2) the velocity of the gas at the instant the projectile is ejected from the gun equals the velocity of the projectile $v_{A.a}$; 3) we take into account the change of momentum ω_{kg} of the gases when the mean rate of motion drops from $U_A = \frac{v_A}{2}$ at the start of discharge to zero

($U = 0$) at the end of the period of after-action.

In such a case it may be assumed that

$$\int_0^{t_n} R dt = \zeta_0 s \int_0^{t_n} p dt + \frac{\omega}{g} \int_{\frac{v_{д.a}}{2}}^{U=0} dU = \zeta_0 s \int_0^{t_n} p dt - \frac{\omega}{g} \frac{v_{д.a}}{2};$$

$$\zeta_0 = (k + 1) x_{cr.}; \quad \zeta_0 = 1.22 - 1.24.$$

p is the mean gas pressure in the bore of the barrel.

The dependence of p on t is expressed by the formula:

$$p = \frac{p_д}{(1 + B't)^{2k/k-1}}, \text{ where } B' = \frac{k - 1}{2} K_0 \frac{s}{\omega} \sqrt{\frac{p_д \Delta}{\Lambda_д + 1}}.$$

Then

$$\int_0^{t_n} R dt = \zeta_0 s p_д \int_0^{t_n} \frac{dt}{(1 + B't)^{2k/k-1}} - \frac{\omega}{g} \frac{v_{д.a}}{2}.$$

Integrating, we get:

$$\int_0^{t_n} R dt = \frac{\zeta_0 s p_д}{B'} \int_0^{t_n} \frac{B' dt}{(1 + B't)^{2k/k-1}} - \frac{\omega}{g} \frac{v_{д.a}}{2} =$$

$$= \frac{\zeta_0 s p_{д}}{B'} \frac{k - 1}{k + 1} \left[1 - \frac{1}{(1 + B't_n)^{k+1/k-1}} \right] - \frac{\omega}{g} \frac{v_{д.а}}{2}.$$

Substituting the value of B', reducing, and bearing in mind that the expression in square brackets can be assumed to be equal to unity (0.995), we get

$$\int_0^{t_n} R dt = \frac{\zeta_0 2\omega p_{д}}{K_0 (k + 1)} \sqrt{\frac{\Lambda_{д} + 1}{p_{д}\Delta}} - \frac{\omega}{g} \frac{v_{д.а}}{2}.$$

Introducing here the values

$$\zeta_0 = (k + 1) \left(\frac{2}{k + 1} \right)^{k/k-1} \quad \text{and } K_0 = \left(\frac{2}{k + 1} \right)^{(1/2)(k+1/k-1)} \sqrt{gk},$$

and multiplying the numerator and denominator of the first term by $\sqrt{gk}$ and bearing in mind that

$$p_{д} \sqrt{\frac{\Lambda_{д} + 1}{p_{д}\Delta}} = \sqrt{p_{д} \frac{W_{KH}}{\omega}} = \sqrt{p_{д} w_{д}} \quad \text{and} \quad \sqrt{gk p_{д} w_{д}} = c_{д},$$

where $c_{д}$ is the speed of sound in a gas under conditions corresponding to the start of discharge, we get:

$$\int_0^{t_n} R dt = \frac{2\omega(k + 1) \left(\frac{2}{k + 1} \right)^{k/k-1} c_{д}}{kg \left(\frac{2}{k + 1} \right)^{(1/2)(k+1/k-1)} (k + 1)} - \frac{\omega}{g} \frac{v_{д.а}}{2} =$$

$$= \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2} \frac{\omega}{g} c_{\text{Д}} - \frac{\omega}{g} \frac{v_{\text{Д}.a}}{2}.$$

Inserting the obtained expression in formula (95), we get:

$$\frac{Q_0}{g} V_{max} = \frac{Q_0}{g} V_{\text{Д}} + \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2} \frac{\omega}{g} c_{\text{Д}} - \frac{\omega}{g} \frac{v_{\text{Д}.a}}{2}.$$

Inasmuch as

$$V_{\text{Д}} = \frac{q + 0.5\omega}{Q_0} v_{\text{Д}.a},$$

$$V_{max} = \frac{q + 0.5\omega}{Q_0} v_{\text{Д}.a} + \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2} \frac{\omega}{Q_0} c_{\text{Д}} - \frac{\omega}{Q_0} \frac{v_{\text{Д}.a}}{2}.$$

Presenting V_{max} in the form

$$V_{max} = \frac{q + \beta\omega}{Q_0} v_{\text{Д}} = \frac{q}{Q_0}\left(1 + \beta \frac{\omega}{q}\right) v_{\text{Д}},$$

we get:

$$V_{max} = \frac{q}{Q_0}\left\{1 + \frac{\omega}{q}\left[\frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2} \frac{c_{\text{Д}}}{v_{\text{Д}}}\right]\right\} v_{\text{Д}},$$

where the coefficient $\beta = \frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2} \frac{c_{\text{Д}}}{v_{\text{Д}}}$ is called the coefficient of gas after-action on the gun mount and depends in the main on the value $c_{\text{Д}}/v_{\text{Д}}$. Since $c_{\text{Д}}$ varies within narrow limits, the predominating

effect on β is produced by the initial (muzzle) velocity of the projectile.

At $k = 1.2$ we get:

$$\beta = 1.59\ c_Д/v_Д,$$

where

$$c_Д = \sqrt{gkp_Д w_Д} = 10.85\ \sqrt{\frac{p_Д(\Lambda_Д + 1)}{\Delta}};$$

at $k = 1.25$

$$\beta = 1.51\ \frac{c_Д}{v_Д};\ c_Д = 11.06\ \sqrt{\frac{p_Д(\Lambda_Д + 1)}{\Delta}}.$$

These formulas tie in the coefficient β with the charging conditions and with the design data. In addition to this theoretical formula, there are also empirical formulas for the β coefficient, for example:

$$\beta_1 = \frac{1400}{v_Д\ \mathrm{m/sec}} + 0.15 \text{ or } \beta_2 = 1300/v_Д.$$

All of these formulas point at the predominating effect of the projectile velocity at the instant of its ejection from the gun.

Example. A 76-mm cannon, $p_Д = 600\ \mathrm{kg/cm^2} = 60{,}000\ \mathrm{kg/dm^2}$; $\Lambda_Д + 1 = 10.0$; $\Delta = 0.70$; $v_Д = 6800$ dm/sec.

$$\beta = 1.59 \cdot 10.85\ \sqrt{\frac{60000 \cdot 10}{0.70}}\ \frac{1}{6800} = 17.26\ \frac{926}{6800} = 2.35.$$

Using the empirical formulas, we get:

$$\beta_1 = \frac{1400}{680} + 0.15 = 2.21; \quad \beta_2 = \frac{1300}{680} = 1.912.$$

It can be seen that the numbers obtained by means of the empirical formulas are smaller than those obtained by the theoretical one.

At a high speed $v_A = 1000$ m/sec, $\Lambda_A + 1 = 5.0$, $\Delta = 0.72$, $p_A = 120{,}000$ kg/dm^2 we get:

$$\beta = 17.26 \sqrt{\frac{120000 \cdot 5}{0.72}} \frac{1}{10000} = 17.26 \frac{913}{10000} = 1.575;$$

$$\beta_1 = \frac{1400}{1000} + 0.15 = 1.40 + 0.15 = 1.55; \quad \beta_2 = \frac{1300}{1000} = 1.30.$$

The value of β_1 closely approaches that of β. If in the first case $Q_0 = 80q$, then at $\omega/q = 0.16$

$$V_{max} = \frac{1}{80}(1 + 2.35 \cdot 0.16) \cdot 680 = \frac{1.376}{80} 680 = 11.7 \text{ m/sec};$$

in the second case $Q_0 = 150q$; $\omega/q = 0.50$ and

$$V_{max} = \frac{1000}{150}(1 + 1.575 \cdot 0.50) = 11.92 \text{ m/sec}.$$

4. THE AFTER-ACTION OF GASES ON A PROJECTILE

The action of gases on the projectile after it leaves the barrel is as follows. The velocity of the gases discharged in the wake of the projectile exceeds that of the projectile, and the gases

surround and overtake the latter, so that the projectile actually moves for a certain period of time within a mobile medium. At the same time the gases continue to exert a pressure on the base of the projectile, thus increasing its velocity even after the projectile has left the bore.

Thus the maximum velocity is not the muzzle velocity, but the velocity at some point a short distance ahead of the gun muzzle face. Nevertheless all the known methods of computation in internal ballistics permit to conclude the computations at the muzzle face. The muzzle velocity v_0 obtained on the basis of test data is computed by reducing the velocity v_c recorded by the chronograph to that of the muzzle face, under the assumption that the velocity beyond the muzzle face is continuously decreased by the action of air resistance.

GRAPHIC NOT REPRODUCIBLE

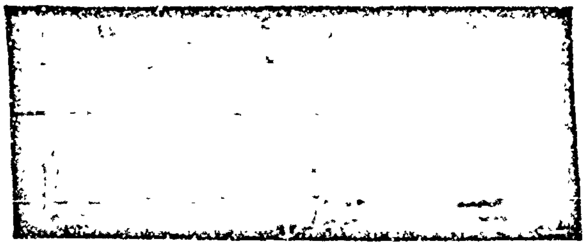

Fig. 115 - Projectile Velocity in the After-Action Period

I) frame-target; II) frame-target.

Figure 115 characterizes the relation between the velocities of the projectile at different points of its trajectory.

The projectile leaves the muzzle face with an absolute velocity $v_{A.a}$ which increases to v_{max} during the period of after-action,

following which it decreases because of air resistance. Using a chronograph and two frame-targets, v_c (the average velocity between frame-targets I and II) is determined. Then, using formula

$$v_0 = v_c + \Delta v_c,$$

where

$$\Delta v_c = \frac{iqX_c}{d^2 \Delta D(v)},$$

we can compute the so-called "initial" velocity of the projectile, whereby the effect of the after-action of the gas powders is not taken into consideration, and the correction Δv_c is computed using the normal resistance law. In consequence we get the following relations:

$$v_0 > v_{max} > v_{A.a}; \quad v_A = v_{A.a} + V_A > v_{A.a}; \quad v_A \approx v_0.$$

Therefore, it may be assumed in practice that the relative muzzle velocity v_A, calculated in solving the problem of internal ballistics, is approximately equal to the "initial" velocity v_0 of the projectile determined by test by means of a chronograph.

The law governing the change of velocity and pressure in a stream of discharged gases, as well as the law governing the change of the stream's shape when the projectile is situated in the stream, lend themselves to experimental analysis only with great difficulty.

Gas dynamics offers only an approximate relationship for determining the gas velocity in the absence of solids distorting the stream, which relationship does not take into account the external pressure.

In view of the absence of reliable theoretical relations for determining the velocity increment of the projectile, we are presenting here certain test data on gas after-action. Spark photography and ultra high-speed photography make it possible to study the phenomena occurring during the motion of the projectile after it has left the barrel and during the discharge of the gases from the bore.

We shall not attempt to enumerate here all the tests of this type and limit our discussion to the firing of small arms and howitzers. When a shot is fired, the air present in the bore is ejected causing a spherical impact wave at the face of the muzzle. Next there appears a small quantity of gas escaping through the clearances between the walls of the bore and the surface of the bullet or projectile, following which there appears the bullet or projectile itself.

Next in order is the discharge of powder gases causing a shock wave upon encountering the outside air, which is responsible for the report of the gun.

The powder gases surround the bullet or projectile and tend to move forward with a velocity considerably exceeding the velocity of the bullet.

Air resistance and friction cause the powder gases to rapidly lose their velocity. A certain distance away from the muzzle face (about 35 cm) the bullet begins to overtake the gases, and the ballistic or bow wave usually accompanying the flight of the projectile

originates at this instant. The photographs in figs. 116, 117, 118 and 119(*) taken by D.F. Chernyshev show that in addition to the ballistic wave around the bullet, a large number of similar waves accompanies the unburned flying particles of powder ejected from the bore.

When the projectile velocity exceeds the speed of sound, the ballistic wave gradually emerges from the spherical sound wave in the form of a cone (see figs. 116, 117, 118 and 119). This is accompanied by clearly defined masses of condensed gas accompanied by eddies and by the appearance of stationary waves when the powder gases are discharged. The latter phenomenon is explained by the following: As the pressure drops gradually, the gases in receding from the muzzle face cause local increases of pressure (pressure jumps), the pressure becoming maximum at the points where the gases become condensed. As the gases are discharged, the position of the first maximum changes - it is gradually displaced toward the muzzle face. The occurrence of such masses of condensed gas is mainly explained by the gradually increasing effect of air resistance.

As the pressure changes, the gas velocity increases at first, mainly at the center of the stream, where the gas is not affected by outside friction; however, at the points of condensation and increased pressure, the gas velocity again undergoes a considerable decrease.

(*) These photos are missing from the original text. Editor.

According to observations made by Kampe-de Ferier in firing a 37-mm cannon having a muzzle velocity of about 720 m/sec, approximately 0.0015 sec before the shell leaves the bore, poorly luminous gases begin to appear from the bore and disperse with a velocity of about 300 m/sec. Directly after the shell leaves the muzzle face, a lateral gas discharge occurs through an annular clearance between the walls of the bore and the base of the projectile with a velocity of about 2000 m/sec. Then, as soon as the base of the projectile recedes from the muzzle face, the expanded gases proceed forward with a velocity of the order of 1400 m/sec, and inasmuch as this velocity greatly exceeds the velocity of the projectile (720 m/sec), the entire gas mass catches up with and overtakes the projectile, and completely surrounds it.

The velocity of the forward layers of the gas mass begins to decrease thereby in the following order:

t, sec.........	0.001	0.002	0.003	0.004	0.005	0.007	0.009
v, m/sec.......	780	750	580	470	370	320	310

The velocity of the projectile continues to approach the value of 720 m/sec, and at t = 0.007 sec it emerges from the gas mass and is relieved of its influence, its distance from the muzzle face being 5 m at that instant.

The gas cloud explodes about 0.019-0.028 sec later, at which time the velocity of the forward layers of the gas increases from 120 to 180 m/sec.

Tests were conducted by Okosi in Japan in 1913 to determine the change in the velocity of a rifle bullet. A special chronograph was

used in these tests permitting the use of several targets simultaneously.

It was found that in 10 cases out of 14 (71%) the velocity was maximum; in the remaining cases (29%) a minimum was observed followed by a maximum, where the maximum velocity exceeded the muzzle velocity in all cases. Okosi concluded that for the Japanese rifle of 1898 issue the maximum velocity is obtained at a distance of about 1.5 m from the muzzle face, and that the increment constitutes only about 0.8% on the average. At a distance of 5 m, the velocity again drops to that of the muzzle velocity.

Tests were conducted by N.M. Platonov for the purpose of determining the period of gas after-action on the base of the projectiles in howitzers with relation to the distance traversed. Curves were obtained showing the change in projectile velocity and the pressure acting on the projectile's base (fig. 120a and b) during the period of after-action. The curves were obtained by means of slow-motion photography.

Figure 121 represents a curve of the pressure exerted on the base of the projectile for a reduced change, obtained from the analysis of the v, X curve.

A comparison of the v, X curves obtained with a full and reduced charge (fig. 120a) disclosed that the length of the period of after-action is approximately doubled in changing from a reduced to a full charge. Curve p_{CH}, x (fig. 121) shows that the pressure exerted by the powder gas on the base of the projectile during the period of after-action rapidly decreases as the distance traversed by the projectile increases.

It should be noted here that in line with the positive results cited here, tests conducted by other investigators employing different methods have produced opposite results. It may be concluded therefore that the subject problem is still in the stage of experimental study and that most of the attention should be directed towards the development of new methods for the study of bullet motion during the period of after-action and for the establishment or determination of errors peculiar to the different methods used.

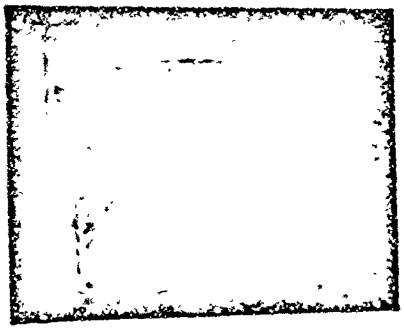

Fig. 120 - Change of Projectile Velocity During the Period of After-Action

a) v, m/sec; b) full charge; c) reduced charge.

Fig. 121 - Pressure Drop During the Period of After-Action

a) kg/cm^2; b) reduced charge.

CHAPTER 3 - BURNING OF POWDER IN AN INCOMPLETELY CLOSED SPACE

1. PRESSURE EXERTED BY GASES WHEN DISCHARGED THROUGH A NOZZLE DURING BURNING OF THE POWDER

The process considered here applies in practice to the following:

1) When powder is burned in a special manometric bomb with nozzle, for the purpose of investigating the burning under conditions simulating powder burning in a gun - where the pressure rises and drops;

2) In a separate combustion chamber of a gas-actuated gun;

3) In the chamber of a rocket shell.

In all of the above cases the gas inflow due to the burning of powder is simultaneously accompanied by the discharge of a portion of the gas through a nozzle. Therefore, the pressure during the burning process may drop as well as increase. Under these conditions the process will vary, depending on the pressure maintained in the chamber: the lower the gas pressure in the chamber, the easier it is to keep it constant. We shall first consider the case of high pressures, for which the burning rate law $u = u_1 p$ is valid.

Let us compile a formula for determining pressure in constant volume W_0 at a given instant under the condition that a portion of the gas is discharged through an opening (nozzle).

We shall designate the quantity of gas discharged at a given instant (in kg) by Y and the ratio $\frac{Y}{\omega} = \eta$.

We had derived above the formula for gas discharge in time t:

$$Y = \int_0^t G_{sec}\, dt,$$

where $G_{sec} = Asp_1$; the pressure in the space from which the gas is discharged equals p_1 and the cross section of the nozzle is s; A is the discharge coefficient.

The pressure p_1 is not constant when burning occurs in a bomb with a nozzle; it changes continuously and hence the discharge process will not be a stable one. In order to evaluate the process, let us assume as a first approximation that the relation derived for a stabilized discharge process also applies to our case, wherein p

varies constantly with respect to time. Then, denoting by p the current gas pressure in a chamber with a nozzle and by s_m the cross section of the nozzle, we get:

$$G_{sec} = As_m p;$$

$$Y = As_m \int_0^t pdt = As_m I$$

and at the end of powder burning

$$Y_K = As_m I_K.$$

We arrive at the conclusion that the gas discharge during burning of the powder is proportional to the pressure increase impulse at the given instant, and at the end of burning - to the full impulse

$$I_K = \frac{e_1}{u_1}.$$

Inasmuch as the impulse I_K depends only on thickness $2e_1$ and the rate of burning u_1, the gas discharge does not depend on the shape of the powder and its burning progressivity. It may be assumed for sufficiently small cross sections s_m and $\tau = \frac{T}{T_1} \approx 1.$(*) In such a case the expression depicting the pressure at a given instant will be:

(*) The relations taking into account the lowering of gas temperature in the presence of large openings are analyzed in special texts.

$$p = \frac{f(\omega\psi - Y)}{W_0 - \alpha(\omega\psi - Y) - \frac{\omega}{\delta}(1-\psi)} = \frac{f\omega(\psi - \eta)}{W_0 - \frac{\omega}{\delta}(1-\psi) - \alpha\omega(\psi - \eta)} =$$

$$= \frac{f\Delta(\psi - \eta)}{1 - \frac{\Delta}{\delta}(1-\psi) - \alpha\Delta(\psi - \eta)}. \tag{96}$$

At the end of burning, we will have:

$$\psi_K = 1, \quad \eta_K = \frac{Y_K}{\omega} = As_m \frac{I_K}{\omega};$$

$$p_K = \frac{f\omega(1 - \eta_K)}{W_0 - \alpha\omega(1 - \eta_K)} = \frac{f\Delta(1 - \eta_K)}{1 - \alpha\Delta(1 - \eta_K)}. \tag{97}$$

Using the designation $\Delta(1 - \eta_K) = \Delta_K$, the formula will be transformed into the usual Noble formula:

$$p_K = \frac{f\Delta_K}{1 - \alpha\Delta_K}, \tag{98}$$

where Δ_K is that charging density at which the maximum pressure $p_m = p_K$ would obtain in a closed space.

The simple rule for calculating the powder charge or the density of the charge producing the required pressure p_K at the end of burning follows from the above. Using Noble's formula, the values are found of Δ_K or ω_K at which the pressure $p_m = p_K$ would obtain in a closed space:

$$\Delta_K = \frac{p_K}{f + \alpha p_K} \quad \text{or} \quad \omega_K = \frac{W_0 p_K}{f + \alpha p_K}.$$

Then, using formula

$$Y_K = As_m I_K = As_m \frac{e_1}{u_1}$$

the weight is determined of the gases discharged through a nozzle of cross section s_m during the period that the powder is burned with impulse I_K. The sum of $\omega_K + Y_K$ will give the full charge which, when burned in a bomb with nozzle s_m, will produce pressure p_K.

$$\omega_1 = \omega_K + Y_K; \quad \Delta_1 = \frac{\omega_1}{W_0}.$$

The value of $I_K = \int_0^1 pdt$ can be found beforehand from a test in a closed bomb, inasmuch as the magnitude of the pressure impulse for powders of simple shapes does not depend on Δ and should not depend on whether the pressure increases according to the law applicable to a closed bomb, or decreases more slowly, or even drops in consequence of the discharge of a portion of the gas through the nozzle. Indeed, if we designate the pressure in a closed bomb by P, and that in a bomb with a nozzle by p, and the times τ and t, respectively, then upon burning powder of the same thickness in a closed space, $de = u_1 Pd\tau$. When powder of the same thickness is burned in a chamber with an opening, $de = u_1 pdt$. Because as a result of partial gas discharge $p < P$, the time interval necessary for the burning of the same thickness de at the smaller pressure p will be correspondingly longer, and the total time will therefore be

$$Pd\tau = pdt,$$

and hence

$$\int_0^1 P d\tau = \int_0^1 p dt = I_K.$$

Therefore, in order to determine the gas flow through the nozzle during the time the powder is burned, use can be made of the impulse $I_K = \int_0^1 P d\tau$ calculated from a test in a closed space (a manometric bomb).

Once the values of I_K and Y_K are known and the value of $I = \int_0^\Psi p dt$ is obtained from the pressure curve, the value of η can be found for any given instant and the corresponding value of ψ then determined.

$$Y = \eta\omega = Y_K \frac{I}{I_K} \text{ or } \eta = \eta_K \frac{I}{I_K}.$$

Solving formula (96) with respect to ψ, we get:

$$\psi = \frac{p\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) + \eta(f + \alpha p)}{f + p\left(\alpha - \frac{1}{\delta}\right)} = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p} + \alpha - \frac{1}{\delta}} + \frac{f + \alpha p}{f + \left(\alpha - \frac{1}{\delta}\right)p}\eta.$$

The first term of this equation represents the usual expression for the portion ψ of the charge at which, in a closed space at the same charging density Δ as in a chamber with a nozzle, the obtained

pressure is p; the second term takes into account the influence of the discharged gases.

The magnitude η_K is precisely the one characterizing the relative intensity of gas flow, or the relative gas discharge during burning of the powder; it is the greater, the greater s_m and I_K and the smaller the charge ω, but always $\eta_K < 1$.

2. PRESSURE CURVE OBTAINED IN A NOZZLED CHAMBER WHEN THE DISCHARGE OPENING IS SMALL

We shall consider, as in the case of general pyrostatics, the case of a powder with a constant burning area ($\varkappa = 1$, $\lambda = 0$, $\sigma = 1$). The instantaneous pressure is expressed by the formula:

$$p = \frac{f\omega(\psi - \eta)}{W_0 - \frac{\omega}{\delta}(1 - \psi) - \alpha\omega(\psi - \eta)} = \frac{f\omega(\psi - \eta)}{W_\psi + \alpha\omega\eta}.$$

The denominator in the right side shows that if a portion of the gas ($\omega\eta$) is discharged, the free space during burning will be greater and hence undergo a smaller change than W_ψ - the free space obtained during burning in a closed space. Hence, as in the case of general pyrostatics, the mean value of the free space can be used to determine the general character of the phenomenon. Assuming that $\psi_{av.} = \frac{1}{2}$ and $\eta_{av.} = \frac{\eta_K}{2} = \frac{As_mI_K}{2\omega}$, we get the following expression for the average value of the free space in the chamber:

$$W_{av.} = W_{\psi av.} + \alpha\omega\eta_{av.} = W_0 - \alpha'\omega + \frac{\alpha}{2}\omega\eta_K.$$

The pressure formula will take on the form:

$$p = \frac{f\omega}{W_{av.}}\,(\psi - \eta).$$

Differentiating with respect to t and bearing in mind that for a powder with a constant burning area or for a strip $\varkappa\sigma_{av.} = 1$ and

$$\frac{d\psi}{dt} = \frac{u_1}{e_1}\,p = \frac{p}{I_K}, \quad \frac{d\eta}{dt} = \frac{As_m p}{\omega},$$

we will get:

$$\frac{dp}{dt} = \frac{f\omega}{W_{av.}}\left(\frac{1}{I_K} - \frac{A}{\omega}s_m\right)p = \frac{f\omega}{W_{av.}I_K}\left(1 - \frac{As_m I_K}{\omega}\right)p =$$

$$\frac{f\omega}{W_{av.}I_K}(1 - \eta_K).$$

Denoting the constant

$$\frac{f\omega}{W_{av.}I_K}(1 - \eta_K) = \frac{1}{\tau_1},$$

separating the variables and integrating the obtained equations we get:

$$\ln\frac{p}{p_B} = \frac{t}{\tau_1}$$

or

$$p = p_B e^{t/\tau_1}.$$

Fig. 122 - Pressure Increase Curves when Burning Powder in a Bomb with a Nozzle.

We have obtained the same formula as when burning powder in a constant closed space, although the constant τ_1 corresponds to the burning of charge $\omega(1 - \eta_K) < \omega$ rather than charge ω, and hence the process of pressure increase is slower, i.e., the same as it would have been in a closed space at $\Delta_K = \frac{\omega(1 - \eta_K)}{W_0}$ which is smaller than the actual $\Delta = \frac{\omega}{W_0}$.

The full time of burning under these conditions is determined by the formula:

$$t_K = 2.303\,\tau_1 \log \frac{p_K}{p_B}.$$

It should be noted that also the pressure p_B of the igniter under conditions of partial gas discharge will not be equal to the rated pressure under conditions of a closed space; a correction must be made for the gas discharge.

The curves in fig. 122 show the pressure increase: 1 - when the powder is burned in a closed space; 2 - when the same charge is burned with a portion of the gas discharged through a nozzle.

Both curves are theoretical ones under the assumption that burning proceeds according to the geometric law. It has been shown however that the true characteristic of pressure increase differs from the theoretical by the fact that the pressure curve is bent at the end and that it approaches the horizontal tangentially rather than at an angle.

Therefore, when powder is burned in a chamber with a nozzle, curve 2 will likewise be distorted.

The problem dealing with the effect of the charging conditions and of the burning of powder on the law governing the pressure increase when a portion of the gas is discharged through a nozzle, can be solved graphically in its first approximation.

Indeed, the input of gases per second as a result of powder burning at high pressures is expressed by the well-known formula:

$$\omega \frac{d\psi}{dt} = \omega \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 p \quad \text{kg/sec} = \omega \Gamma p;$$

and the gas discharge (output) per second is expressed by the following formula:

$$G_{sec} = \frac{dY}{dt} = \omega \frac{d\eta}{dt} = As_m p.$$

If the input exceeds the output, the pressure in the chamber will rise; if the procedure is reversed, the pressure will drop.

If the input equals the output, the pressure must be maximum or remain constant. Hence the pressure change depends on the ratios $d\omega/dt$ and $d\eta/dt$. A simple relationship is obtained between the charging conditions and the powder burning characteristics, permitting a direct answer to the problems dealing with the nature of the pressure curve and with the condition of obtaining the maximum pressure before the end of burning.

Everything depends on the ratios:

$$\omega\Gamma = \omega \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1 \text{ and } As_m,$$

where As_m, characterizing the flow conditions, is a constant, and $\omega\Gamma$ is usually a variable and becomes constant only for powders with a constant burning area. Therefore, these values can be made equal and the pressure p_{max} can be obtained only at a certain instant. Thereafter the pressure will begin to drop or rise because S/S_1 usually varies in one direction only.

We thus get a simple graphic solution for the problem. If it is required to find out whether the end of burning will obtain after the maximum p_{max} is reached and the value of ψ_m to which the maximum pressure will correspond, the answers will be obtained by constructing a curve for the given powder depicting the progressivity of burning $\omega\Gamma$ as a function of ψ, and a straight line aa' must be then drawn parallel to the abscissa at a distance As_m from it (fig. 123). If

the entire line aa' lies below line 1-1, the gas input during the entire burning process exceeds the gas discharge through the nozzle, the pressure curve rises continuously, and the maximum pressure coincides with the end of burning. The angle of inclination of the curve $\left(\frac{dp}{dt}\right)_K$ will be maximum at the end of burning (curve 1-1 in fig. 124). If line aa' starts below the $\omega\Gamma$ curve (see fig. 123) and then intersects it at point b and continues above it, it means that the pressure will first increase, pass the maximum at point $b(\psi_m)$ where the gas input equals the gas discharge, and will then drop, because the gas discharge As_m per second will exceed the gas input $\omega\Gamma$.

A pressure curve 2-2 (fig. 124) is thus obtained with a smooth inflection at point p_m and a descending portion showing a pressure drop between p_m and p_K.

In fig. 123 curve Γ_T (3-3) lies below the line aa'. This indicates that the discharge will always exceed the input, that the pressure will continuously decrease (curve 3-3 in fig. 124), and that the powder may burn slowly and may even tend to die out.

The Γ curves in fig. 123 represent strip powders of varying thicknesses: 1-1 for thin strip, 2-2 for strips of average thickness, and 3-3 for thick powder. Hence, with the same powder shape, by varying the thickness of the strip and leaving the charge and cross-sectional area of the nozzle unchanged, we can obtain all three forms of the pressure (increase) curve.

Contrariwise, for the same powder of given dimensions, by varying the nozzle opening s_m or the weight of the charge ω, the position of line aa' or $\omega\Gamma$ can be changed and with it the characteristic of

the curve depicting the pressure increase in a chamber with a nozzle. Therefore, a chamber with a nozzle, if provided with means for recording the rise and drop of pressure, makes it possible to test powders at considerably greater charging densities and under conditions approaching those of powder burned in a weapon; i.e., not only under conditions of pressure rise, but also under conditions of pressure drop.

All of the above conclusions and the possibility of obtaining maximum pressure before the end of burning are based on the analysis of theoretical curves of progressivity Γ calculated according to the geometric law. Actually, of course, the test curves Γ, ψ differ in character, i.e., they differ in regard to regressive powders by their beginning and end portions, whereas in regard to progressive powders they differ along their entire zero-to-unity interval. Typical diagrams for strip (or tubular) powder (fig. 125) and for powders with many perforations (fig. 126) are presented below.

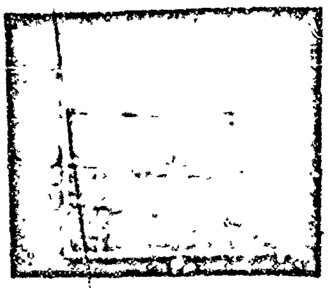

Fig. 123 - Gas Input and Discharge Characteristic Curves.

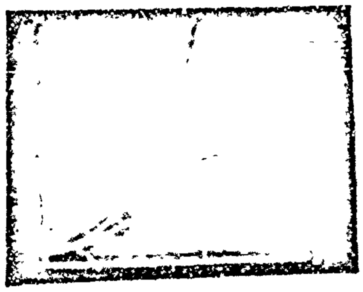

Fig. 124 - Curves Depicting Pressure Variation in a Bomb with a Nozzle.

A comparison of the $\omega\Gamma_{On}$ and aa' diagrams characterizing the

size of the nozzle indicates the important difference between experimental and theoretical curves Γ, ψ which must be reflected on the nature of the pressure curves p, t obtained in the presence of a nozzle on the chamber.

Inasmuch as the test curves Γ, ψ show a sharp drop at the end of burning and tend toward zero, they must be intersected by the line aa'; the maximum must occur before the end of burning, whereas the end of burning will occur on the descending branch of the pressure curve.

The ascending portions of the Γ, ψ curves at the start of burning point at a gradual ignition at ignitor pressures of 20-40 kg/cm^2, and if the Γ, ω curve lies a considerable distance below the corresponding straight line aa', ignition cannot take place and the powder will be extinguished because of lowered pressure. Such examples were obtained in testing cylindrical grains for ignition. When a pyroxyline igniter was used capable of developing a pressure of about 50 kg/cm^2, it was often found that after it was burned its gases would exit through the nozzle without igniting the powder charge. A subsequent examination of the powder grains would show that the latter were partly burned and became extinguished when the pressure dropped. Actual calculations for one such case show that

$$A \frac{s_m}{\omega} = 0.007 \frac{0.07}{0.005} = 0.098 \text{ cm}^2 \text{ kg} \cdot \text{sec},$$

and for this $\frac{7}{7}$ powder the theoretical $\Gamma_0 = 20 \cdot 0.0075 = 0.150$ $cm^2/kg \cdot sec$.

GRAPHIC NOT REPRODUCIBLE

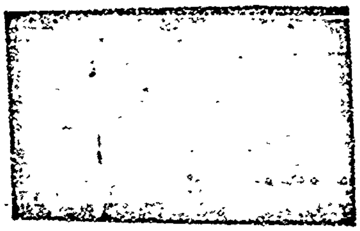

Fig. 125 - Relation Between Gas Input and Discharge per Second when Burning Strip Powder in Accordance with the Physical Combustion Law.

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Fig. 126 - Relation Between Gas Input and Discharge per Second when Burning Powder with Many Perforations in Accordance with the Physical Combustion Law.

If the ignition were instantaneous, the powder would not have been extinguished because $\Gamma_{T.O.} > A \frac{s_m}{\omega}$(*). But inasmuch as ignition is not instantaneous, and the initial Γ can actually equal 0.040-0.050 and then increase to 0.200, the value of Γ at the start of ignition is actually smaller than $A \frac{s_m}{\omega}$, the discharge through the nozzle exceeds the gas input, and the powder does not ignite.

(*) Subscript T.O. stands for "theoretical, initial." Editor.

3. DERIVATION OF MAXIMUM PRESSURE FORMULA

According to the physical law of burning, all powders without exception, when burned in a chamber with a nozzle, due to the sharp surface area decrease at the end of burning, must develop maximum pressure before the end of burning, and hence a maximum $\frac{dp}{dt} = 0$ must occur on the pressure curve without fail. We shall derive the condition for obtaining maximum pressure and a formula for p_m, from the fundamental equation of pressure in a semi-closed space.

We had presented above the general pressure formula:

$$p = \frac{f\Delta(\psi - \eta)}{1 - \frac{\Delta}{\delta}(1 - \psi) - \alpha\Delta(\psi - \eta)} = \frac{a}{b}.$$

In order to determine the conditions for obtaining p_m, we differentiate p with respect to t, bearing in mind that

$$\frac{d\psi}{dt} = \Gamma p \text{ and } \frac{d\eta}{dt} = A\frac{s_m}{\omega}p.$$

We get:

$$p = \frac{f\Delta\left(\frac{d\psi}{dt} - \frac{d\eta}{dt}\right)}{b} - p\,\frac{\left[-\alpha\Delta\left(\frac{d\psi}{dt} - \frac{d\eta}{dt}\right) + \frac{\Delta}{\delta}\frac{d\psi}{dt}\right]}{b} =$$

$$= \left\{ \frac{f\Delta\left(\Gamma - A\,\frac{s_m}{\omega}\right) - p\left[\frac{\Delta}{\delta}\,\Gamma - \alpha\Delta\Gamma + \alpha\Delta\,A\,\frac{s_m}{\omega}\right]}{b} \right\} p.$$

Equating the derivative to zero, we obtain the condition necessary for obtaining p_m:

$$f\Delta\left(\Gamma_m - A\,\frac{s_m}{\omega}\right) - p_m\left[\alpha\Delta A\,\frac{s_m}{\omega} - \Delta\Gamma_m\left(\alpha - \frac{1}{\delta}\right)\right] = 0.$$

Eliminating Δ and dividing by f, we reduce similar terms:

$$\Gamma_m\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] = A\,\frac{s_m}{\omega}\left(1 + \alpha\,\frac{p_m}{f}\right),$$

whence

$$\omega\Gamma_m = As_m n', \qquad \text{(N)}$$

where

$$n' = \frac{1 + \alpha\frac{p_m}{f}}{1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}} > 1.$$

At $f = 900{,}000$, $\alpha = 1$; the value of n' depends on pressure p_m and can be computed in advance:

$$n'_{200} = 1.014; \quad n'_{1000} = 1.066; \quad n'_{2000} = 1.126.$$

In the first approximation for rockets, where $p_m \leqslant 250$ atm, $n' = 1$; for chambers with nozzles $n' = 1.10$.

Thus, in order to obtain p_m, it is necessary that the inflow of gas per second at $p = 1$, i.e., $\omega\Gamma$, satisfy the condition (N), i.e., that it exceed somewhat the discharge of gas per second reduced to $p = 1$.

$$\Gamma_m = \frac{As_m}{\omega} n' \quad \text{or} \quad \omega\Gamma_m > As_m.$$

Having obtained from bomb tests or by means of theoretical calculations curves of ω and Γ as a function of I, and also the straight lines $\frac{As_m}{\omega} n'$ for various charges ω_i, the problem of determining p_m can be solved as follows.

Passing the straight line $A \frac{s_m}{\omega_i} n'$ for a given weight of charge through curve Γ, I in fig. 127, we find the point of intersection a_i, and dropping a vertical from this point we determine I_{mi} on the abscissa and the value ψ_{mi} on curve ψ. With $A\frac{s_m}{\omega_i}$ and I_{mi} known, we compute the relative gas discharge at the instant p_m is obtained:

$$\eta_{mi} = A \frac{s_m}{\omega_i} I_{mi}.$$

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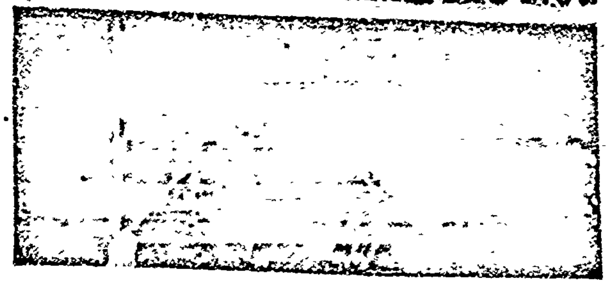

Fig. 127 - Graph for Determining p_m During Gas Discharge.

Substituting the obtained values of ψ_m and η_m in the pressure formula, we find:

$$p_m = \frac{f\Delta(\psi_m - \eta_m)}{1 - \frac{\Delta}{\delta}(1 - \psi_m) - \alpha\Delta(\psi_m - \eta_m)},$$

where

$$\eta_m = Y : \omega = \frac{As_m I}{\omega}.$$

It is known that the gas inflow $\psi = \int_0^I \Gamma dI$ is expressed by the area bounded by the Γ curve and the abscissa I, and the gas discharge $\eta = \frac{As_m}{\omega} I$ by the area of a triangle of altitude $A\frac{s_m}{\omega}$ and base I. The difference between these areas, cross-hatched in fig. 127, gives the gas residue in the chamber. The greater the charge ω_1, the lower will be the straight line $A\frac{s_m}{\omega_1}$, the greater will be the cross-hatched area $\psi_m - \eta_m$, the later will the maximum pressure occur, and the greater will be the maximum pressure.

It is of interest to note that the condition for obtaining maximum pressure obviously does not depend on the volume of the chamber and the charging density, but, rather, on the ratio between the oppositely reacting intensities of gas inflow $\omega\Gamma_m$ and gas discharge As_m, similarly to the condition in a gun wherein the maximum obtainable pressure depends on the ratio between the intensity of gas inflow $\omega\Gamma$ and the rate of increase of the volume of the bore sv.

The obtained derivations are valid for high pressures (above 1000 kg/cm^2) or for rapidly burned fine powders, when the burning rate law $u = u_1 p$ holds true.

At low pressures (up to 250 kg/cm^2) and for very thick powders the burning rate law $u = u_1 p$ no longer applies, as was shown in the chapter dealing with the burning rate law, and the relationships become somewhat different.

At small pressures and relatively slow burning of the powder, the mass of the latter succeeds in becoming heated to a considerable degree; the more so, the slower the process of burning. Therefore the rate of burning u_1 reduced to $p = 1$ increases at low pressures, and begins to decrease as the pressure increases.

Inasmuch as the true change of u_1 with heating and the degree to which the powder mass becomes heated at different rates of burning have not yet been determined experimentally, formally this phenomenon of burning reduced to $p = 1$, which is more intense at small pressures and less intense at high pressures, can be expressed by the burning rate law:

$$u = u_1' p,$$

where $u_1' > u_1$, whereby

$$u_1 = \frac{u_1'}{p^{1-\nu}}.$$

In such a case the rate of gas inflow $\omega \frac{d\Psi}{dt}$ will be expressed by the formula:

$$\omega \frac{d\Psi}{dt} = \omega \frac{S_1}{\Lambda_1} \frac{S}{S_1} u_1' p^{\nu} = \delta S u_1' p^{\nu},$$

and the intensity of discharge will be expressed by the previous relation

$$G_{sec} = \frac{dY}{dt} = A s_m p.$$

As was shown by Prof. Ya. M. Shapiro, this diversity of the exponents in the laws governing the input and discharge of the gases leads to a very interesting property of self-regulation and leveling-off of the p_m value, manifested during the burning of thick powders in bombs with nozzles at low pressures (10 to 200 kg/cm^2).

Indeed, if we were to depict the input and discharge of gases in fig. 128, the first process would be represented by a parabolic curve and the second process by a straight line passing through the origin of the coordinates, whose tangent equals $A\frac{s_m}{\omega}$ and can be chosen

at will.

Say, at point a the input and discharge of the gases become equalized and the pressure remains constant. Should the pressure be increased (to the right of point a), the intensity of gas discharge will become greater compared with the gas input and the pressure will drop, i.e., the process will reverse itself towards point a, maintaining p_m = const.

In exactly the same way, when the pressure drops (to the left of point a), the gas inflow process will be more intense, and this will cause the pressure to increase and to tend towards p_m = const.

Therefore, at low pressures, when the burning rate law is $u = u_1' p^{\nu}$, the process of maintaining the gas pressure at a specific level will be of the self-regulating kind; it will be more stable compared with the process of pressure change when powder is burned at high pressures of the order of 1000-2000 kg/cm^2.

This tendency towards leveling off of the pressure can be noted by comparing diagrams Γ, I and $A\frac{s_m}{\omega}$, I under different burning rate laws. It has been established by actual tests that at high pressures, at $\Delta >$ 0.10-0.12, the integral curves I as a function of ψ do not depend on Δ and coincide at different charging densities. At small charging densities and low pressures the integral curves assume lower positions, which are the lower, the smaller the charging density.

Correspondingly, the Γ, ψ and I, ψ curves also coincide at high charging densities; at low charging densities the Γ, ψ curves are disposed the higher, the smaller the value of Δ; curves Γ, I, at the start, are likewise disposed higher and are then intersected by

Γ, I curves at higher values of Δ, because the total area $\int_0^{i_K} \Gamma dI = 1 = const.$

GRAPHIC NOT REPRODUCIBLE

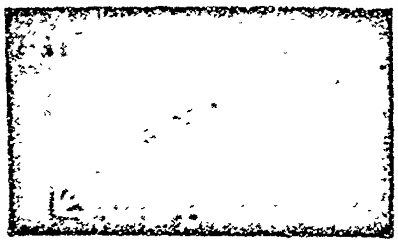

Fig. 128 - Diagram Depicting the Rate of Input and Discharge of Gases.

For high densities we will have the former graph (see fig. 127), where Γ, I and ψ, I are the same for different Δ(from 0.12 to 0.25). We shall construct an additional graph (fig. 129) for low charging densities taking into account the change of Γ and I obtained with the change of the charging density. Let $\Delta_1 < \Delta_2 < \Delta_3$; let us see what happens when the gas inflow with velocity Γ occurs simultaneously with a gas discharge at the rate of $\frac{As_m}{\omega}$ per second.

GRAPHIC NOT REPRODUCIBLE

Fig. 129 - Rate of Gas Inflow and Discharge at Small Values of Δ

The smaller the value of Δ, the smaller will be the pressure developed by burning powder in a constant closed space, and the

higher will be the disposition of the Γ, I curve on the graph.

When the input Γ_m balan s the discharge $A\frac{s_m}{\omega}n'$ at point a_3, the straight line $\frac{As_m}{\omega}n'$ will lie above Γ_3 and the pressure will begin to drop; however, at a lower pressure, use must be made of curve Γ_2 lying above Γ_3, and point a_2 can be intercepted at the same pressure. The same will occur in region a_2-a_1, whereby $\psi_3 < \psi_2 < \psi_1$ - the burned portion of the charge grows, whereas the pressure remains constant, because the gas inflow equals the gas discharge. This will not occur at all at high pressures, at which curve Γ, I is the same even after it intercepts point a_1, point a_2 or point a_3. The Γ, I curve will be disposed below line $\frac{As_m}{\omega}n'$, and the pressure will continue to drop only.

K.E. TSIOLKOVSKY'S FORMULA

A rocket is propelled by the reaction force produced by the gases discharged from it. The Great Fatherland War has given us many examples of rocket application both in our country and in the countries of our allies and enemies. These may be exemplified by our famous "Katushas" or by the German multi-barreled rocket (mine) throwers.

We shall present here the derivation of the famous Tsiolkovsky formula for determining the velocity of a rocket on the basis of the relations presented above.

We shall designate by Q the total weight of the rocket, the charge included, by ω - the weight of the powder charge, and by q - the weight of the rocket less the charge, so that $Q = q + \omega$.

The total weight of the rocket will vary in flight from Q to q. Say, the weight of the gases discharged at a given time is Y_{kg}. The equation of quantity of motion (momentum) will be:

$$\frac{Q - Y}{g} dv = Rdt - \zeta s_m pdt - \zeta s_m dI,$$

but

$$dY = Gdt = As_m dI;$$

eliminating $s_m dI$, we get:

$$\frac{Q - Y}{g} dv = \frac{\zeta}{A} dY;$$

$$dv = \frac{\zeta g}{A} \frac{dY}{Q - Y} = - \frac{\zeta g}{A} \frac{d(Q - Y)}{Q - Y};$$

$$v = \frac{\zeta g}{A} \ln \frac{G}{Q - Y} = \frac{2.303\zeta g}{A} \log \frac{Q}{Q - Y}.$$

The greatest velocity will obtain after all the gas is discharged from the combustion chamber, i.e., $Y_{max} = \omega$:

$$Y_{max} = \frac{2.303\zeta g}{A} \log \frac{Q}{Q - \omega} \approx 32300 \log \frac{Q}{Q - \omega} \quad \text{dm/sec.}$$

This is Tsiolkovsky's formula derived without taking air resistance into consideration.

The values of ζ depending on the degree of expansion of the nozzle were presented above. The table given below gives these values with respect to the ratio between the discharge diameter d_a of the nozzle and its smallest cross section d_m (Table 32).

Table 32

$\frac{d_a}{d_m}$	1	2	2.5	3	4
ζ	1.24	1.61	1.675	1.73	1.80

Therefore, at $d_a/d_m = 2$ the gain in the reaction force produced compared with a straight nozzle is 30%. When d_a/d_m is increased to 3 and 4, the added increase in the reaction force amounts to only 7 and 4%, respectively.

Inasmuch as the enlargement of the discharge cross section of the nozzle is associated with increase of length and weight, which add to the weight of the rocket without offering any appreciable advantage, the value of d_a/d_m in actual practice is taken within the limits of 2-2.5.

CHAPTER 4 - A BRIEF DISCUSSION OF THE THEORY OF THE MUZZLE BRAKE

1. GENERAL CONSIDERATIONS

A muzzle brake is a device attached to the muzzle of the barrel. Its purpose is to deflect a portion of the discharged gases in the direction of the barrel recoil and thus reduce the velocity of recoil and the load imposed on the gun mount.

A portion of the gas entering the muzzle brake moves in the direction (behind) the projectile through the center opening of the brake, and the other portion of gas is discharged through side openings of the brake in the direction of recoil.

The deflection of the gases to the sides reduces the quantity of gas passing through the center opening of the brake behind the projectile, and this serves to reduce the maximum velocity of recoil. The reaction produced by a portion of the gases discharged through the side openings creates a force counteracting the power of recoil and also retards the latter.

Thus the main purpose of a muzzle brake is to reduce the energy of the recoiling parts.

Introducing the designations:

V_{max} - maximum velocity of free recoil without muzzle brake;

V_T - velocity of free recoil at the end of gas after-action in the presence of a muzzle brake,

then the efficiency of the muzzle brake may be called the "relative reduction of the kinetic energy of the recoiling masses," i.e.,

$$\gamma' = \frac{V_{max}^2 - V_T^2}{V_{max}^2}.$$

The corresponding relative reduction of the maximum velocity of recoil can be denoted thus:

$$r = \frac{V_{max} - V_T}{V_{max}},$$

whereby

$$\gamma' = r(2 - r).$$

If the difference between the weights of the recoiling parts in the presence of the muzzle brake (Q_T) and in the absence of the latter (Q_0) is taken into account:

$$\gamma' = \frac{v^2_{max} - \frac{Q_T}{Q_0} v^2_T}{v^2_{max}}.$$

The simplest types of muzzle brakes are the "active brakes," whose action is based on the impact of gases escaping in the wake of the projectile against a surface fastened in front of the barrel (fig. 130).

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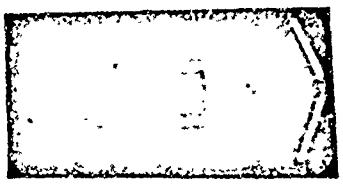

Fig. 130 - Diagram of an Active Brake.

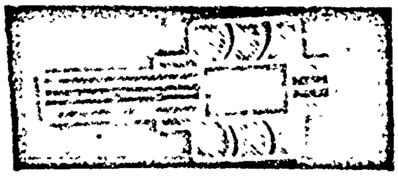

Fig. 131 - Diagram of a Reaction Brake.

In reaction brakes the gases are discharged through curved passageways. The change of momentum along the bore axis will be equal to the reaction impulse of the stream against the deflecting brake surface (fig. 131).

2. GAS REACTION PRESSURE ON THE WALLS OF A CURVED BORE OF A MUZZLE BRAKE(*)

Let us not consider the flow of gas through a curved bore (fig. 132)

(*) D.A. Ventsel, "VNUTRENNIAYA BALLISTIKA" (Internal Ballistics) Part II, 1939.

whose entrance cross section is F_1 and exit cross section is F_2. The passageways are inclined at an angle a_1 with respect to the bore axis at the entrance opening and at an angle a_2 at the exit opening.

We shall apply the equation of the change of momentum to the volume of gas bounded by the curved walls of the bore and the two sections F_1 and F_2 normal to it.

The mass of gas entering the bore through section F_1 during the time interval dt is $\frac{G_T}{g}$ dt, whose component of the momentum along the axis of the gun barrel equals

$$\frac{G_T}{g} U_1 \cos \alpha_1 dt.$$

The same gas mass $\frac{G_T}{g}$dt will exit through section F_2, and will have a component of the momentum along the same axis equal to

$$\frac{G_T}{g} U_2 \cos \alpha_2 dt.$$

The increment of the projection of the momentum of the given volume on the x-axis equals the elementary impulse of time dt along the x-axis of the total pressure exerted by the bore on the gas and the pressures in the sections normal to it.

Inasmuch as the component of the pressure exerted by the bore on the gas along the x-axis equals the component of the gas reaction R_T on the bore with its sign reversed, we can write

$$\frac{G_T}{g}(U_2 \cos \alpha_2 - U_1 \cos \alpha_1)dt = -R_{Tx}dt + F_1 p_1 \cos \alpha_1 dt - F_2 p_2 \cos\alpha_2 dt,$$

whence, bearing in mind that $\alpha_2 > \frac{\pi}{2}$ and $\cos \alpha_2 < 0$,

$$R_{Tx} = \frac{G_T}{g}(U_2 \left|\cos \alpha_2\right| + U_1 \cos \alpha_1) + F_2 p_2 \left|\cos \alpha_2\right| + F_1 p_1 \cos \alpha_1. \tag{99}$$

GRAPHIC NOT REPRODUCIBLE

Fig. 132 - Diagram of Forces Acting in the Bore of the Muzzle Brake.

It follows that the component of the gas reaction on the bore along the x-axis is positive; it counteracts the recoil.

If the brake has several bores or passages inclined at the same angles α_1 and α_2, the expression for the reaction of the whole brake will remain exactly the same, where the designations R_{Tx}, G_T, F_1, F_2 relate to the sum of the areas of all the passages in the brake.

3. TOTAL REACTION R_x ON THE GUN BY GASES DISCHARGED THROUGH THE MUZZLE BRAKE

A gun equipped with a muzzle brake is subjected to the following forces acting along its axis during gas discharge.

1) The component along the x-axis of the reaction of gases discharged through the forward end of the brake (muzzle opening of the

barrel)

$$R_x = -(k+1)\left(\frac{2}{k+1}\right)^{k/k-1} sp = \zeta sp.$$

2) The component along the x-axis of the reaction of gases flowing into the muzzle brake through section F_1

$$R_{Tx} = -\frac{G_T}{g} U_1 \cos \alpha_1 - F_1 p_1 \cos \alpha_1 = -\left(\frac{G_T}{g} U_1 + F_1 p_1\right) \cos \alpha_1.$$

3) The component along the x-axis of the gas reaction on the passageways of the muzzle brake (formula 99)

$$R_{Tx} = \frac{G_T}{g}(U_2 \left|\cos \alpha_2\right| + U_1 \cos \alpha_1) + F_2 p_2 \left|\cos \alpha_2\right| + F_1 p_1 \cos \alpha_1.$$

We shall assume that the gas begins to flow simultaneously through all the openings after the base of the projectile has passed through the muzzle face. The component of the total reaction along the x-axis will be

$$R_{\Sigma x} = -\zeta sp + \left(\frac{G_T}{g} U_2 + F_2 p_2\right) \left|\cos \alpha_2\right|.$$

The first term is greater than the second, and the minus sign in front of the first term indicates that the total gas reaction on the gun acts in a direction opposite to that of the x-axis (opposite to the direction of the projectile's motion). The closer angle α_2 approaches π, the greater the values of G_T and F_2 and the greater the

reaction force of the brake.

Obviously, the entry angle α_1 to the passageways in the brake does not enter into the expression for the total reaction R_Σ.

In computing the amount G_T and the velocity U_2 of the discharge from the passageways, we shall use the assumption that the pressure p_1 at the entrance to the brake passageways is critical with relation to the mean pressure in the bore of the gun at a given instant:

$$p_1 = x_{cr.}p = \left(\frac{2}{k+1}\right)^{k/k-1} p;$$

the incoming gas velocity U_1 may be disregarded.

By expanding in succession the values in R_Σ, Prof. D.A. Ventsel reduced this expression to the following general form:

$$R_\Sigma = \alpha_\Sigma \zeta s p,$$

where

$$\alpha_\Sigma = 1 - \frac{1}{k+1}\left[\Psi\chi \frac{2k}{\sqrt{k^2-1}}\left(\frac{2}{k+1}\right)^{1/k-1}\sqrt{1-\left(\frac{p_2}{p_1}\right)^{k-1/k}} + \frac{F_2 p_2}{F_1 p_1}\right]\frac{F_2}{s}\left|\cos\alpha_2\right|.$$

In the last expression:

χ - a coefficient depending on the curvature of the passageways (χ = 0.75-1.0 at $\alpha_1 < 30^o$);

Ψ - a coefficient depending on the entrance angles α_1 and α_2 in terms of expression $\frac{\alpha_1 + \chi - \alpha_2}{2}$; its value is given in a table.

Table 33

$\frac{F_2}{F_1}$	$\frac{p_2}{p_1}$
1.01	0.5
1.06	0.4
1.19	0.3
1.46	0.2
2.25	0.1
3.61	0.05
11.8	0.01

The ratio of the pressures at the entrance and exit of the brake bore p_2/p_1 depends on the ratio F_2/F_1 and is determined from Table 33.

$\frac{\alpha_1 + \pi - \alpha_2}{2}$	15^o	20^o	25^o	30^o	40^o	50^o	60^o	70^o
Ψ	0.725	0.770	0.815	0.845	0.890	0.920	0.940	0.950

4. THE FULL IMPULSE OF THE TOTAL GAS REACTION

Similarly to a gun without a muzzle brake, we will have the following during the period of after-action between the gases and the brake:

$$\frac{Q_0}{g}(V_T - V_\Delta) = \int_0^{t_n} R_\Sigma dt - \frac{\omega}{g}\frac{V_{A.a}}{2} = I_\Sigma - \frac{\omega}{g}\frac{V_{A.a}}{2},$$

whence

$$V_T = V_A + \frac{g}{Q_0} I_\Sigma - \frac{\omega}{Q_0} \frac{v_{A.a}}{2},$$

where

$$B_\Sigma = \alpha_\Sigma \zeta s p.$$

The pressure drop $\bar{p}$ as a function of time will be the same as in the usual case (in the absence of a muzzle brake), with the exception that the coefficient B' is replaced by the greater coefficient B_Σ, because the gases are discharged not only through the front opening of the brake, but also through the side passages:

$$p = \frac{p_A}{(1 + B_\Sigma t)^{2k/k-1}},$$

where

$$B_\Sigma = B' \left[1 + \chi \frac{F_1}{s} \left(\frac{2}{k+1} \right)^{(1/2)(k+1/k-1)} \right].$$

The period of after-action in the presence of a muzzle brake is

$$t_{пг} = \frac{1}{B_\Sigma} \left[\left(\frac{2}{k+1} \right)^{1/2} \left(\frac{p_A}{p_a} \right)^{k-1/2k} - 1 \right].$$

Assuming $R_\Sigma = \alpha_\Sigma \xi sp$, substituting this expression for the impulse I_Σ and integrating, we get:

$$I_\Sigma = \alpha_\Sigma \xi sp_A \int_0^{t_n} \frac{dt}{(1 + B_\Sigma t)^{2k/k-1}} =$$

$$= \alpha_\Sigma (k - 1)\left(\frac{2}{k + 1}\right)^{k/k-1} \frac{sp_A}{B_\Sigma}\left[1 - \frac{1}{(1 + B_\Sigma t)^{k+1/k-1}}\right],$$

where

$$\xi = (k + 1)\left(\frac{2}{k + 1}\right)^{k/k-1};$$

$$B_\Sigma = B'\left[1 + \chi \frac{F_1}{s}\left(\frac{2}{k + 1}\right)^{(1/2)(k+1/k-1)}\right];$$

$$B' = \frac{k - 1}{2}\left(\frac{2}{k + 1}\right)^{(1/2)(k+1/k-1} \frac{\sqrt{gkp_A w_A}}{l_0 + l_A}.$$

The function in brackets is close to unity.

Upon substituting $I_\Sigma - \frac{\omega}{g}\frac{v_{A..a}}{2}$ and $V_A = \frac{q + 0.5\omega}{Q_0} v_A$ in the expression for V_T, we get the final expression:

$$V_T = \frac{q}{Q_0}\left(1 + \frac{1}{2}\frac{\omega}{q}\right) v_A + \alpha_\Sigma \frac{2}{k}\left(\frac{2}{k + 1}\right)^{1/2} \frac{\omega}{Q_0} c_A -$$

$$-\frac{\omega}{Q_0}\frac{v_{Д.a}}{2}=\frac{q}{Q_0}\left(1+\beta_{\Sigma}\frac{\omega}{q}\right)v_Д,$$

where

$$\beta_{\Sigma}=\alpha_{\Sigma}\frac{2}{k}\left(\frac{2}{k+1}\right)^{1/2}\frac{c_Д}{v_Д}.$$

At $k=1.2$

$$\beta_{\Sigma}=1.589\alpha_{\Sigma}\frac{c_Д}{v_Д}\text{ and }c_Д=10{,}85\sqrt{\frac{p_Д(1+\Lambda_Д)}{\Delta}}.$$

Here p is in kg/cm^2, the velocity is in m/sec, Δ is in kg/dm^3.

Using the above formula, we can calculate the velocity V_T at the end of the period of gas after-action on the barrel and determine the efficiency of the brake:

$$\text{efficiency }\eta=\frac{V_{max}^2-V_T^2}{V_{max}^2}=1-\left(\frac{1+B_{\Sigma}\frac{\omega}{q}}{1+\beta\frac{\omega}{q}}\right)^2.$$

The efficiency of modern muzzle brakes may be of the order of 40-50% and even 70 and 80% in exceptional cases.

Example. Calculate the efficiency of a muzzle brake. Say, the characteristics of the given gun are as follows:

$\Delta=0.72$; $\frac{\omega}{q}=0.453$; $\Lambda_Д=4.63$; $v_Д=1000$ m/sec; $p_Д=983$ kg/cm^2

(subscript A represents muzzle-Translator).

In the absence of a muzzle brake (at $k = 1.2$)

$$\beta = 1.59 \frac{c_A}{v_A} = \frac{1.59}{v_A} \sqrt{gkp_A \frac{\Lambda_A + 1}{\Delta}} = \frac{1.59}{1000} \sqrt{117.7 \cdot 983 \cdot \frac{5.63}{0.72}} =$$

$$= 1.59 \frac{950}{1000} = 1.510.$$

$$1 + \beta \frac{\omega}{q} = 1 + 1.510 \cdot 0.453 = 1.684.$$

In the presence of a muzzle brake

$$\beta_\Sigma = 1.59\alpha_\Sigma \frac{c_A}{v_A} = \alpha_\Sigma \beta;$$

$$\alpha_\Sigma = 1 - \frac{1}{k+1}\left[\psi\chi \frac{2k}{\sqrt{k^2 - 1}} \left(\frac{2}{k+1}\right)^{1/k-1} \sqrt{1 - \left(\frac{p_2}{p_1}\right)^{k-1/k}} + \right.$$

$$\left. + \frac{F_2}{F_1} \cdot \frac{p_2}{p_1}\right] \frac{F_1}{s} \cos \alpha_2 .$$

Say, the characteristics of the brake are as follows:

$$\alpha_1 = 30^\circ; \quad \alpha_2 = 120^\circ; \quad \frac{F_2}{s} = 1.5; \quad \frac{F_2}{F_1} = 1.01; \quad \frac{\alpha_1 + \chi - \alpha_2}{2} = 45^\circ.$$

According to Table 33, $p_2/p_1 = 0.5$; $\Psi = 0.905$; we shall assume that $\chi = 1$; $|\cos 120^\circ| = 0.50$.

For $k = 1.2$ $\frac{k-1}{k} = \frac{1}{6}$; $\frac{2k}{\sqrt{k^2 - 1}}\left(\frac{2}{k+1}\right)^{1/k-1} = 2.25$;

$$\left(\frac{p_2}{p_1}\right)^{k-1/k} = 0.5^{1/6} = 0.891.$$

$$\alpha_\Sigma = 1 - \frac{1}{2.2}\left[0.905 \cdot 1 \cdot 2.25 \cdot \sqrt{1 - 0.891} + 1.01 \cdot 0.50\right] 1.5 \cdot 0.5 =$$

$$= 1 - \frac{1}{2.2}\left[0.672 + 0.505\right] 0.75 = 1 - 0.40 = 0.60$$

$$\beta_\Sigma = 0.60 \cdot 1.510 = 0.906$$

$$1 + \beta_\Sigma \frac{\omega}{q} = 1 + 0.906 \cdot 0.453 = 1.410$$

$$\eta = 1 - \left(\frac{1 + \beta_\Sigma \frac{\omega}{q}}{1 + \beta \frac{\omega}{q}}\right)^2 = 1 - \left(\frac{1.410}{1.684}\right)^2 = 1 - 0.702 = 0.298.$$

Thus the given brake will absorb about 30% of the energy of free recoil.